Int J Theor Phys (2011) 50: 122–128
DOI 10.1007/s10773-010-0500-5

Geometric Phase for Two Entangled Spin-1/2 Particles in a Magnetic Field

Xue-Hua Yi · Xin-Ding Zhang

Received: 19 July 2010 / Accepted: 17 September 2010 / Published online: 9 October 2010

Abstract In this paper the geometric phases of two entangled spin-1/2 particles in the presence and absence of spin-spin interaction are calculated. We also discuss the geometric phases when only one of the two particles is affected by the external magnetic field. Our results show that the geometric phase in this case is not equal to that of a single particle under the same evolution condition because of the effect of entanglement. We further study the entanglement dependence of the noncyclic geometric phases in the interacting and noninteracting spins under a time-independent uniform magnetic field. A general entanglement-dependence geometric phase is formulated.

Keywords Spin-1/2 particle · Geometric phase · Dynamic phase · Quantum entanglement · Uniform magnetic field

1 Introduction

The concept of geometric phase was first introduced by Pancharatnam [1] in his study of interference between light waves on the distinct states of polarization. Although his treatment was essentially classical, the notion of geometric phase was later shown to have important consequence for quantum system. However, Berry [2] found a geometric phase factor $\gamma_n(c) = i\oint_c \langle n(\vec{R})|\nabla_{\vec{R}}|n(\vec{R})\rangle \cdot d\vec{R}$ ($\vec{R}$ is parameter space, and $\vec{R}(T) = \vec{R}(0)$) in addition to the familiar dynamical phase factor $-\frac{1}{\hbar}\int_0^T E_n(\vec{R})dt$ when a quantum system undergoes a cyclic adiabatic evolution. After Berry's rediscovery, the geometric phase has attracted a great deal of interests. One of the important extension was performed by Aharonov and

X.-H. Yi
School of Physics and Optical Information Sciences, Jiaying University, Guangdong, Meizhou 514015, P.R. China

X.-H. Yi · X.-D. Zhang (✉)
Laboratory of Quantum Information Technology, ICMP and SPTE, South China Normal University, Guangzhou, P.R. China
e-mail: xdzhang2000@gmail.com

Anandan [3]. They generalized the geometric phase from adiabatic approximation to nonadiabatic condition. Whereafter, a series of experimental and theoretical progress were achieved in many fields, such as nuclear physics, optics, quantum information, quantum field theory and so on [4–13]. The geometric phase for a pair of entangled spin-$1/2$ particles under rotating magnetic fields was given in [14]. And very recently the geometric phase for a pair of interacting spin-$1/2$ particles with one particle driven by a slowly rotating magnetic field was investigated [15]. Inspired by the result in [15], here we mainly consider the entanglement dependence of the geometric phase in SU(2) subsystems.

For a noncyclic process, the total phase accumulated during the evolution can be denote as the sum of dynamic and geometric phase, $\gamma_{tot} = \gamma_{dyn} + \gamma_{geo}$, where the dynamic phase and the geometric phase are given as [14, 16, 17]

$$\gamma_{dyn} = -i \int_0^T \langle \Psi(t) \mid \dot{\Psi}(t) \rangle dt = -i \int_0^T \langle \Psi(0) | U(t)^+ \dot{U}(t) | \Psi(0) \rangle, \quad (1)$$

$$\gamma_{tot} = \arg \langle \Psi(0) \mid \Psi(T) \rangle = \arg \{ \langle \Psi(0) | U(t) | \Psi(0) \rangle \}. \quad (2)$$

$$\begin{aligned} \gamma_{geo} &= \gamma_{tot} - \gamma_{dyn} \\ &= \arg \{ \langle \Psi(0) | U(t) | \Psi(0) \rangle \} + i \int_0^T \langle \Psi(0) | U(t)^+ \dot{U}(t) | \Psi(0) \rangle \, dt. \end{aligned} \quad (3)$$

The geometric phase γ_{geo} is real number and usually depends only on the curve Γ in the projective Hilbert space. This "gemetrical" feature is argued to be resilient to certain random noises during quantum gate operations and may lead to promissing applications in quantum information.

2 Noncyclic Geometric Phase

To get a simple picture of geometric phase, we first consider a single spin-$1/2$ particle under a time-independent uniform magnetic field along the z direction. Assume that the initial spin state $|\vec{n}(0)\rangle$ makes an angle θ with the z axis. During noncyclic evolution process, the spin state at time t is written as

$$|\Psi(t)\rangle = e^{-i\varphi(t)/2} \cos \frac{\theta}{2} | \uparrow \rangle + e^{i\varphi(t)/2} \sin \frac{\theta}{2} | \downarrow \rangle, \quad (4)$$

where $\varphi(t) = \varphi(0) + \omega t$, $\varphi(0)$ is the initial phase and ω is the Larmor frequency proportional to the magnetic field strength, as shown in Fig. 1. The curve Γ in the projective Hilbert space is isomorphic to the curve C_n on the Poincaré sphere. Spheric coordinate vector corresponding to the curve C_n is $\mathbf{n}(t) = (\sin\theta \cos\varphi(t), \sin\theta \sin\varphi(t), \cos\theta)$ $(t \in [0, T])$. Inserting (4) into (3), we obtain the noncyclic geometric phase

$$\gamma_{geo}(C_n) = -\arctan\left(\cos\theta \tan\frac{\omega T}{2}\right) + \frac{\omega T}{2} \cos\theta = -\frac{1}{2}\Omega\left[C_n^{g-c}\right], \quad (5)$$

where $\Omega[C_n^{g-c}]$ is the solid angle enclosed by the cure C_n^{g-c} that consists of C_n and the shortest geodesic on Poincaré sphere connecting the end points $\mathbf{n}(0)$ and $\mathbf{n}(T) \neq -\mathbf{n}(0)$. Note that for a given curve C_n, $\gamma_{geo}(C_n)$ is independent of the strength of magnetic field. That's, it depends on the precession angle ωT, but not ω.

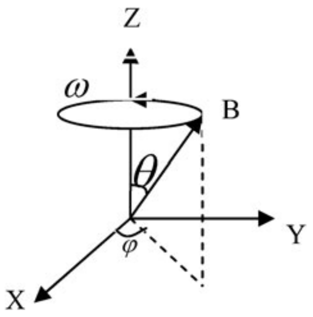

Fig. 1 Vector relation for spin-1/2 in uniform magnetic field

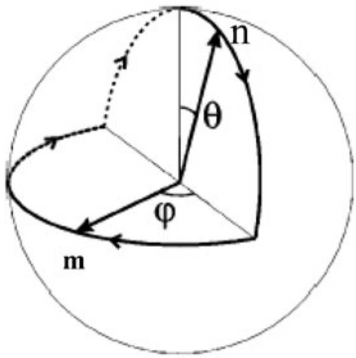

Fig. 2 Poincaré sphere (a closed loop described particle by the polarization vector of the external driving field)

3 Geometric Phase for Two Spin-1/2 Particles Entangled States

According to Schmidt's theorem [18], the state of two spin-1/2 particles may be expressed as:

$$|\Psi\rangle = e^{-i\beta/2}\cos\frac{\alpha}{2}|n\rangle|m\rangle + e^{i\beta/2}\sin\frac{\alpha}{2}|-n\rangle|m\rangle, \tag{6}$$

$|n\rangle$ and $|m\rangle$ is the state vector of particle 1 and 2, n and m are two points on the Poincaré sphere (as shown in Fig. 2). α and β are two Schmidt parameters constants. Random point $\mathbf{p}$ on the Schmidt sphere is given by $\mathbf{p} = (\sin\alpha\cos\beta, \sin\alpha\sin\beta, \cos\alpha)$. So antipodal points correspond to orthogonal states, i.e. $\langle -\mathbf{p}|\mathbf{p}\rangle = 0$. The angle α determines the degree of entanglement state. $\alpha = 0$ or π corresponds to product state. When $\alpha = \pi/2$, the degree of entanglement is maximized.

Consider two entangled spin-1/2 particles that undergo spin precession in an external time-independent uniform magnetic field in the z direction, and assume that the two particles do not interact with each other. The Hamiltonian reads

$$\hat{H} = \omega_1 \hat{S}_{1Z} + \omega_2 \hat{S}_{2Z}, \tag{7}$$

where ω_1 and ω_2 are the Larmor frequencies,and $\hat{S}_{1Z}$ and $\hat{S}_{2Z}$ are the corresponding z components of the spin operators associated with the two particles. The states of the system S belong to the Hilbert space $H_2 \equiv H \otimes H$. Vectors in H_2 can be expanded as Schmidt decompositions. Thus, the evolution of any normalized initial and any time state for the system S processes respectively according to

$$|\Psi(0)\rangle = e^{-i\beta/2}\cos\frac{\alpha}{2}|n(0)\rangle|m(0)\rangle + e^{i\beta/2}\sin\frac{\alpha}{2}|-n(0)\rangle|-m(0)\rangle, \tag{8a}$$

$$|\Psi(t)\rangle = e^{-i\beta/2}\cos\frac{\alpha}{2}|n(t)\rangle|m(t)\rangle + e^{i\beta/2}\sin\frac{\alpha}{2}|-n(t)\rangle|-m(t)\rangle. \tag{8b}$$

Assuming that the initial spin states $|n(0)\rangle$ and $|m(0)\rangle$ is

$$|n(0)\rangle = e^{-i\varphi_1(0)/2}\cos\frac{\theta_1}{2}|\uparrow\rangle + e^{i\varphi_1(0)/2}\sin\frac{\theta_1}{2}|\downarrow\rangle, \tag{8c}$$

$$|m(0)\rangle = e^{-i\varphi_2(0)/2}\cos\frac{\theta_2}{2}|\uparrow\rangle + e^{i\varphi_2(0)/2}\sin\frac{\theta_2}{2}|\downarrow\rangle, \tag{8d}$$

and the arbitrary spin states $|n(t)\rangle$ and $|m(t)\rangle$ are given by

$$|n(t)\rangle = e^{-i\varphi_1(t)/2}\cos\frac{\theta_1}{2}|\uparrow\rangle + e^{i\varphi_1(t)/2}\sin\frac{\theta_1}{2}|\downarrow\rangle, \tag{9a}$$

$$|-n(t)\rangle = -e^{-i\varphi_1(t)/2}\sin\frac{\theta_1}{2}|\uparrow\rangle + e^{i\varphi_1(t)/2}\cos\frac{\theta_1}{2}|\downarrow\rangle, \tag{9b}$$

$$|m(t)\rangle = e^{-i\varphi_2(t)/2}\cos\frac{\theta_2}{2}|\uparrow\rangle + e^{i\varphi_2(t)/2}\sin\frac{\theta_2}{2}|\downarrow\rangle, \tag{9c}$$

$$|-m(t)\rangle = -e^{-i\varphi_2(t)/2}\sin\frac{\theta_2}{2}|\uparrow\rangle + e^{i\varphi_2(t)/2}\cos\frac{\theta_2}{2}|\downarrow\rangle. \tag{9d}$$

Substituting (8) into (1), we get dynamic phase

$$\gamma_{dyn} = -\cos\alpha\left(\frac{\omega_1 T}{2}\cos\theta_1 + \frac{\omega_2 T}{2}\cos\theta_2\right). \tag{10}$$

Substituting (8) and (9) into (2), and utilizing normalized condition, we obtained total phase through complicated calculation

$$\begin{aligned}\gamma_{tot} &= \arg\langle\Psi(0)\mid\Psi(T)\rangle\\ &= -\arctan\left(\frac{\cos\theta_1\tan\frac{\omega_1 T}{2} + \cos\theta_2\tan\frac{\omega_2 T}{2}}{1-(\cos\theta_1\cos\theta_2 + \sin\alpha\cos\beta\sin\theta_1\sin\theta_2)\tan\frac{\omega_1 T}{2}\tan\frac{\omega_2 T}{2}}\right).\end{aligned}$$

So the geometric phase is gotten as

$$\begin{aligned}\gamma_{geo}[\Gamma] = &-\arctan\left(\frac{\cos\theta_1\tan\frac{\omega_1 T}{2} + \cos\theta_2\tan\frac{\omega_2 T}{2}}{1-(\cos\theta_1\cos\theta_2 + \sin\alpha\cos\beta\sin\theta_1\sin\theta_2)\tan\frac{\omega_1 T}{2}\tan\frac{\omega_2 T}{2}}\right)\\ &+\cos\alpha\left(\frac{\omega_1 T}{2}\cos\theta_1 + \frac{\omega_2 T}{2}\cos\theta_2\right).\end{aligned} \tag{11}$$

Where Γ is the path in the projective Hilbert space CP^3. (θ_1,ω_1) and (θ_2,ω_2) are the spherical polar angles and angle frequency of **n** and **m**, respectively. Even if their degree of entanglement is the same, the geometric phase $\gamma_{geo}[\Gamma]$ is different for distinct sates. The dynamical phase for each spin is canceled by entanglement. Thus the inseparability of $\gamma_{geo}[\Gamma]$ in (10) can entirely be traced back to the inseparability of the total phase $\arg\langle\Psi(0)|\Psi(T)\rangle$.

Despite the complexity in the (10) and (11) for the geometric phase in its full generality, we can extract some interesting results. From the two equations, the dynamic phase can be divided into two parts corresponding to the phases of the two separate particles. However, the total phase as well as the geometric phase cannot be separated except under some special cases. The latter case arises primarily because of the entanglement and the interaction of the two particles.

When state vector is at the north ($\alpha = 0$) or south ($\alpha = \pi$) pole of the Schmidt sphere, i.e. entanglement disappear, geometric phase is

$$\begin{aligned}\gamma_{geo}[\Gamma] &= -\arctan\left(\frac{\cos\theta_1 \tan\frac{\omega_1 T}{2} + \cos\theta_2 \tan\frac{\omega_2 T}{2}}{1 - \cos\theta_1 \cos\theta_2 \tan\frac{\omega_1 T}{2} \tan\frac{\omega_2 T}{2}}\right) \pm \left(\frac{\omega_1 T}{2}\cos\theta_1 + \frac{\omega_2 T}{2}\cos\theta_2\right)\\ &= \gamma_{geo}[C_n] \pm \gamma_{geo}[C_m]. \end{aligned} \tag{12}$$

Equation (12) implies that two-particle geometric phase for product states may be analyzed in terms of the geodesically closed solid angle for each spin.

Now assuming that only one of the particles (such as the particle 1) is affected by the external magnetic field, the result shows that the second particle still influence the two-particle geometric phase through entanglement. Let $\omega_2 = 0$ in (11), we obtain the geometric phase for this state

$$\gamma_{geo}[\Gamma] = -\arctan\left(\cos\theta_1 \tan\frac{\omega_1 T}{2}\right) + \frac{\omega_1 T}{2}\cos\alpha\cos\theta_1. \tag{13}$$

Which differs from the one-particle geometric phase $\gamma_{geo}(C_n)$ in (5). Since entanglement interaction exists in this system. Thus, entanglement essentially affects the geometric phase of a system. When decomposition of spin is parallel or antiparallel to the magnetic field, i.e, $\theta_1 = 0$ or $\theta_1 = \pi$. In this case we obtain

$$\gamma_{geo}[\Gamma] = \mp\arctan\left(\tan\frac{\omega_1 T}{2}\right) \pm \frac{\omega_1 T}{2}\cos\alpha. \tag{14}$$

By comparing this result with (5), we find that α play the role of the constant polar angle of the affected spin.

4 Geometric Phase for Two Spin-1/2 Particles with a Spin–Spin Interaction

In this section we consider a quantum system consisting of two spin-1/2 particles with an isotropic spin-spin interaction. The time-independent uniform magnetic field is applied in the z direction. The Hamiltonian operator is

$$\hat{H} = \omega_1 \hat{S}_{1Z} + \omega_2 \hat{S}_{2Z} + (8\lambda/\hbar)(a_x S_x^1 S_x^2 + a_y S_y^1 S_y^2 + a_z S_z^1 S_z^2), \tag{15}$$

where λ and (a_x, a_y, a_z) are the strength of the interaction. When $a_x = a_y = a_z = a = \hbar/4$, i.e The Hamiltonian operator is in the case of the isotropic spin-spin interaction and nonzero field. In this case, we give the Hamiltonian operator $\hat{H}$ in (15) in matrix form

$$\begin{aligned} H &= (\omega_1 \sigma_z \otimes I + \omega_2 I \otimes \sigma_z) + (2\lambda a/\hbar)\left(\sigma_x \otimes \sigma_x + \sigma_y \otimes \sigma_y + \sigma_z \otimes \sigma_z\right)\\ &= \begin{pmatrix} \omega_1 + \omega_2 + \frac{\lambda}{2} & 0 & 0 & 0 \\ 0 & -\frac{\lambda}{2} & \lambda & 0 \\ 0 & \lambda & -\frac{\lambda}{2} & 0 \\ 0 & 0 & 0 & -\omega_1 - \omega_2 + \frac{\lambda}{2} \end{pmatrix}. \end{aligned} \tag{16}$$

The normalized initial state of the quantum system is expressed in the matrix form

$$|\Psi(0)\rangle = \begin{pmatrix} \exp[-i(\beta+\frac{\varphi_1+\varphi_2}{2})](\cos\frac{\alpha}{2}\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}+\sin\frac{\alpha}{2}\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}) \\ \exp[-i(\beta+\frac{\varphi_1-\varphi_2}{2})](\cos\frac{\alpha}{2}\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}-\sin\frac{\alpha}{2}\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}) \\ \exp[i(\beta+\frac{\varphi_1-\varphi_2}{2})](\cos\frac{\alpha}{2}\sin\frac{\theta_1}{2}\cos\frac{\theta_2}{2}-\sin\frac{\alpha}{2}\cos\frac{\theta_1}{2}\sin\frac{\theta_2}{2}) \\ \exp[i(\beta+\frac{\varphi_1+\varphi_2}{2})](\cos\frac{\alpha}{2}\sin\frac{\theta_1}{2}\sin\frac{\theta_2}{2}+\sin\frac{\alpha}{2}\cos\frac{\theta_1}{2}\cos\frac{\theta_2}{2}) \end{pmatrix}. \tag{17}$$

The state vector at any later time t is $|\Psi(t)\rangle = U(t)|\Psi(0)\rangle$, where $U(t)$ is the operator of unitary evolution

$$U(t) = e^{-i\lambda t/2}e^{-iHt} = \begin{pmatrix} e^{-it(\lambda+\omega_1+\omega_2)} & 0 & 0 & 0 \\ 0 & \frac{e^{-i\lambda t}+e^{i\lambda t}}{2} & \frac{e^{-i\lambda t}-e^{i\lambda t}}{2} & 0 \\ 0 & \frac{e^{-i\lambda t}-e^{i\lambda t}}{2} & \frac{e^{-i\lambda t}+e^{i\lambda t}}{2} & 0 \\ 0 & 0 & 0 & e^{-it(\lambda-\omega_1-\omega_2)} \end{pmatrix}. \tag{18}$$

We obtain the total phase by complicated calculation

$$\gamma_{tot} = \arg\langle\Psi(0)\mid\Psi(T)\rangle = \arctan\frac{\mathrm{Im}(T)}{\mathrm{Re}(T)}, \tag{19}$$

where

$$\begin{aligned}\mathrm{Im}(T) &= \cos\alpha\sin(\omega_1+\omega_2)\left(\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}-\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\right) \\ &\quad -\sin\lambda T\left\{\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}+\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}+\frac{1}{2}\sin\alpha\sin\theta_1\sin\theta_2\right. \\ &\quad +\left[\frac{1}{2}\sin\theta_1\sin\theta_2-\sin\alpha\left(\cos^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}+\sin^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\right)\right] \\ &\quad \left.\times\cos 2\left(\beta+\frac{\varphi_1-\varphi_2}{2}\right)\right\}, \\ \mathrm{Re}(T) &= \cos\lambda T+\cos(\omega_1+\omega_2)\left(\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}+\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}+\frac{1}{2}\sin\alpha\sin\theta_1\sin\theta_2\right),\end{aligned}$$

the dynamic phase is

$$\begin{aligned}\gamma_{dyn} &= -i\int_0^T\langle\Psi(t)\mid\Psi(t)\rangle\,dt = -i\int_0^T\langle\Psi(0)|\,U^+(t)\,\dot{U}(t)\,|\Psi(0)\rangle\,dt \\ &= \lambda T\left\{\cos(2\beta+\varphi_1-\varphi_2)\left[\sin\alpha\left(\sin^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}+\cos^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}\right)-\frac{1}{2}\sin\theta_1\sin\theta_2\right]\right. \\ &\quad -\left(\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}+\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\right)-\frac{1}{2}\sin\alpha\sin\theta_1\sin\theta_2 \\ &\quad \left.+(\omega_1+\omega_2)\cos\alpha\left(\sin^2\frac{\theta_1}{2}\sin^2\frac{\theta_2}{2}-\cos^2\frac{\theta_1}{2}\cos^2\frac{\theta_2}{2}\right)\right\}.\end{aligned}$$

Thus we get the geometric phase as

$$\gamma_{geo} = \gamma_{tot}-\gamma_{dyn}. \tag{20}$$

From (20), we see that the geometric phase is closely associated with the strength λT of the spin-spin interaction. This result is really novel and more general compared with other relative research work.

5 Summary

In this paper we study the influence of entanglement on the noncyclic two-particle geometric phase for two different spin-$1/2$ models. We propose a more general formula (20) of entanglement-dependence geometric phase with a spin-spin interaction under a uniform magnetic field. Our formalism is applicable to any entanglement dependence of the geometric phase in the case of SU(2) subsystems. As two special examples, the geometric phases for both a pair of spin-$1/2$ particles with two spin processing and an isotropic spin-spin interaction are reproduced by mean of the present formulas. Futhermore, we derived the geometric phase for an isotropic interaction and nonzero magnetic field using the present formulas. The geometric phase for noninteracting spin-$1/2$ particles processing in an external time-independent magnetic field and prepared in an entangled Schmidt state has been shown to exhibit a rich entanglement dependence. We proposed a general entanglement dependence geometric phase in a spin-spin interaction model and formulated the corresponding unification scheme for that geometric phase. This result should be useful in their applications of quantum information processing [6, 19] and further study of geometric phase [11, 15].

Acknowledgements This work was supported by the Natural Science Foundation of Jiaying University (08KJ31), NTFS No. 10805020, and NSFC under Grant No. 10805020.

References

1. Pancharatnam, S.: Proc. Indian Acad. Sci. A **44**, 247 (1956)
2. Berry, M.V.: Proc. R. Soc. A **392**, 45 (1984)
3. Aharonov, Y., Anandan, J.: Phys. Rev. Lett. **58**, 1593 (1987)
4. Chiao, R.Y., Wu, Y.S.: Phys. Rev. Lett. **57**, 933 (1986)
5. Mead, C.A.: Phys. Rev. Lett. **59**, 161 (1987)
6. Zhu, S.L., Wang, Z.D.: Phys. Rev. Lett. **89**, 097902 (2002)
7. Zhu, S.L., Wang, Z.D.: Phys. Rev. B **61**, 1142–1147 (2000)
8. Pal, D., Basu, B., Bandyopadhyay, P.: Phys. Lett. A **299**, 304 (2002)
9. Li, H.Z.: Chin. Phys. Lett. **19**, 153–155 (2002)
10. Fuentes-Guridi, I., Carollo, A., Bose, S., Vedral, V.: Phys. Rev. Lett. **89**, 220404 (2002)
11. Chen, H.W., Hu, M., Chen, J.L., Du, J.F.: Phys. Rev. A **80**, 054101 (2009)
12. Ma, R.Q., Li, Y.F., Shi, J.: Acta Phys. Sin. **57**, 4083 (2008) (in Chinese)
13. Zhu, S.L., Wang, Z.D.: Phys. Rev. A **67**, 022319 (2003)
14. Tong, D.M., Kwek, L.C., Oh, C.H.: J. Phys. A **36**, 1149 (2003)
15. Yi, X.X., Wang, L.C., Zheng, T.Y.: Phys. Rev. Lett. **92**, 150406 (2004)
16. Pati, A.K.: Phys. Rev. A **52**, 2576–2584 (1995)
17. Abdel-Aziz, H.S.: Commun. Theor. Phys. **42**, 672–674 (2004)
18. Peresl, A.: Quantum Theory: Methods and Concepts. Kluwer Academic, Dordrecht (1995)
19. Feng, X.L., Wu, C., Sun, H., Oh, C.H.: Phys. Rev. Lett. **103**, 200501 (2009)